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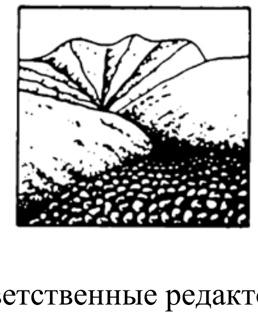

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Mathematical modeling of physical and mechanical processes in the debris flow source area

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Abstract. Mathematical modeling of physical and mechanical processes in the debris flow source area is considered. Material is thought of as medium of inhomogeneous structure medium, which properties vary with time. Two levels are separated. Macrolevel is used for analyzing ground massif as a whole, and there is microlevel where elementary macrovolumes forming the source area are investigated. Heat transfer equation, diffusion equation, and equation of non-homogeneous compression of the beam are solved at the macrolevel. Processes of phase change, evaporation-condensation, mechanical failure, and deformation are calculated at the microlevel. Typical size of basic macrovolumes is much greater than molecular-kinetic sizes, and it is much less than the distance where macroscopic parameters, such as Young's modulus, heat-conduction coefficient, density of material, considerably change. Results of simulation can be utilized for building prediction formulae for debris flow forecast.

***Key words**: analysis, mathematical modeling, forecast, debris flow source area, debris flow, inhomogeneous structure medium, physical and mechanical process*

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Математическое моделирование физико-механических процессов в селевых очагах

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Аннотация. Рассматривается математическое моделирование физико-механических процессов в селевых очагах. Материал представляется как структурно-неоднородная среда с изменяющимися во времени свойствами. Выделены макроуровень для расчёта массива пород как единого неоднородного тела, и микроуровень, на котором анализируются элементарные макрообъёмы, формирующие зону зарождения селей. На макроуровне решаются уравнения диффузии, теплопроводности и неоднородного сжатия, а на микроуровне рассчитываются процессы фазового перехода, испарения-конденсации, механического деформирования и разрушения. Характерный размер элементарных макрообъёмов много больше молекулярно-кинетических размеров, но много меньше расстояний, на которых существенно меняются макроскопические параметры, такие как модуль Юнга, коэффициент теплопроводности и плотность вещества. Результаты имитации могут быть использованы для создания прогностических зависимостей для предсказания селей.

***Ключевые слова:** анализ, математическое моделирование, прогноз, селевой очаг, сель, структурно-неоднородная среда, физико-механический процесс*

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Introduction

Some of the most dangerous slope processes taking place in the mountains and foothills are debris flows [*Anakhaev et al., 2012*]. In most cases, these natural phenomena result in enough negative effect on natural landscapes, obstructions and destructions in human settlements, harm of utility facilities, and loss of life [*Anakhaev et al., 2012*]. Therefore, developing methodologies of their forecast presences some features of interest.

Various physical and mechanical processes happen in the debris flow source area. For example, water supplying for glacial debris flow is provided by melting of glacier [*Fleishman, 1978*], so phase change takes place. Besides, fluid movement occurs in ground massif. Material temperature may variate too, having, for instance, a large impact on thaw of buried ice. And, without question, no slope process arises if there is no mechanical failure. Moreover, these things do not exist in isolation but have a significant influence on each other.

As a matter of fact, formation of friable fragmental rock in origination sites, movement of debris flow, accumulation of its deposits would be considered not as a hydrological or gravitational process, but as a phenomenon of a more complex nature [*Kazakov, 2015*]. Nonlinear structures with chaotic behavior containing an unlimited number of elements, which are debris flow geosystems, can be described as self-organizing objects in which self-organization of ordered components occurs sequentially [*Kazakov, 2015, Kyul et al., 2012*].

The following approach is used for a comprehensive analysis of origin and movement of debris flows. Material is thought of as medium of inhomogeneous structure medium, which properties vary with time. Two levels are separated. Macrolevel is used for analyzing ground massif as a whole, and there is microlevel where elementary macrovolumes forming the source area are investigated. Heat transfer equation, diffusion equation, and equation of non-homogeneous compression of the beam are solved at the macrolevel. Phase change, evaporation-condensation, mechanical failure, and deformation are calculated at the microlevel. All processes influence each other. Typical size of basic macrovolumes is much greater than molecular-kinetic sizes, and it is much less than the distance where macroscopic parameters considerably change [*Sokolkin et al., 1984*].

Properties of material and, accordingly, coefficients of differential equations describing the debris flow source as a whole are calculated on the base of parameters of elementary macrovolumes. At the same time, solutions of these equations affect elementary macrovolumes.

Independently such an approach is poorly suited for debris flow forecasting in real time, because risk of obtaining inadequate results in solving non-linear problems takes place in each case, and computer costs may be unreasonable. But it provides to get training set of acceptable size suitable for building prediction formulae.

Design diagram of the debris flow source area and solution of time-dependent problem

It is supposed that debris flow source area consists of spherical particles of rock and ice connected with cylinder bonds, water, and wet air. As time passes, these elements may destroy or change.

System of heat transfer equation, diffusion equation, and equation of non-homogeneous compression of the beam is solved at the macrolevel. Step-by-step method is utilized.

It is known that both large and small step sizes have drawbacks in terms of accuracy. A large step size may introduce significant truncation errors and reduce accuracy, using a small step size may lead to serious round-off errors [*Rzhanitsin, 1982*]. Therefore, optimization of this step is a part of analysis. It is performed in the following way.

The problem is solved, utilizing various steps. In the course of computing, number of changes of sign of first order derivative of solution and response results to decreasing of step are recorded. After that, dependences of these parameters on step are built, and their minimums are calculated. The method of ordered risk minimization [*Vapnik, 2006*] is used.

It is difficult to check of results of calculation, utilizing full-scale experiment, because works in the debris flow source area are complicated and may be unsafe. However, effects detected with the help of mathematical modeling may be confirmed by published data, and it is possible to estimate accuracy of debris flow forecast according to prediction formulae developed by the means of training set obtained by simulation of physical and mechanical processes in ground massif.

For instance, analysis shows chance of occurring of infrasound torsion vibrations in the debris flow source area before descent. A certain extent, it gets in line with results of solution of momentum theory of elasticity problem for seismic centre [*Belonosov, 1998*]. Infrasonics may well initiate abnormal behavior of a wide variety of animals and run to great distances [*Khorbenko, 1986*]. Such activity before debris flow was demonstrated by grass snakes [*Ershov, 1979*], domestic animals, and poultry [*Marikovskiy, 1984*].

Other example is estimation of slope inclination of zone of debris flow deposition. Computed value amounts to 1.9°, which is close to known quantity of 2° [*Fleishman, 1978*].

The third example is development of prediction formulae, using training set calculated with help of mathematical modeling, for Gerkhozhan tract (Kabardino-Balkarian Republic, Russia).

According to [*Zimin et al., 2001*] volume of loose materials is more than 10^8 m^3, thickness of moraine equals to 40–50 m, buried ice depth amounts to 4–30 m, volume of buried ice comes out at $3.8 \cdot 10^7$ m^3.

This debris flow origination zone has the following granulometric texture by volume [*Zimin et al., 2001*]:

1) ratchel having particle sizes of more than 20 mm (45–64%);
2) gritrock consisting of particle with sizes from 2 to 20 mm (10–15%);
3) sand being made up of particles with sizes from 0.05 to 2 mm (15–25%);
4) silt consisting of particle with sizes from 0.002 to 0.05 mm (4–5%);
5) clay particles having sizes of less than 0.002 mm (7–10%).

Developed mathematical support simulated activity of group of experts. Thus, prediction formulae knowingly have semantic charge, which increases effectiveness of operation.

The following factors are initially calculated:

$$p_{t10} = \begin{cases} \frac{2}{\pi} arctan(0.00012 t_{10}^{3.76}) if t_{10} \geq 0 \\ -\frac{2}{\pi} arctan[0.00012(-t_{10})^{3.76}] if t_{10} < 0 \end{cases}, \quad (1)$$

where $\mathbf{p_{t10}}$ is factor taking into account influence of average temperature for the last 10 days on possibility of debris flow descent, $\mathbf{t_{10}}$ is average temperature for the last 10 days, °C;

$$p_t = \begin{cases} p_{t10} if p_{t10} \leq \frac{2}{\pi} arctan(3.2 \cdot 10^{-10} t^{3.7}) \\ \frac{2}{\pi p_h} arctan(3.2 \cdot 10^{-10} t^{3.7}) if \\ p_{t10} > \frac{2}{\pi} arctan(3.2 \cdot 10^{-10} t^{3.7}) \end{cases}, \quad (2)$$

where $\mathbf{p_t}$ is factor making allowance for influence of sum of average daily temperatures after average daily temperature to exceed zero on possibility of debris flow descent, **t** is sum of average daily temperatures after average daily temperature to exceed zero, °C;

$$p_q = \begin{cases} 0 if (t_{10} < 0°C and t_s < 0°C) and q < 52mm \\ \frac{2}{\pi} arctan(0.000686 q^{2.46}) if (t_{10} \geq 0°C or t_s \geq 0°C) or q \geq 52mm \end{cases}, (3)$$

where $\mathbf{p_q}$ is factor adverting influence of daily precipitation amount on possibility of debris flow descent, $\mathbf{t_s}$ is average daily temperature, **q** is daily precipitation amount, mm;

$$p_{oq} = \begin{cases} 0 if\ (t_{10} < 0°C and t_s < 0°C) and q < 52mm \\ \frac{2}{\pi} arctan(0.032 \cdot o) if (t_{10} \geq 0°C or t_s \geq 0°C) or q \geq 52mm \end{cases}, \quad (4)$$

where $\mathbf{p_{oq}}$ is factor allowing influence of average precipitation intensity for the last 3 h on possibility of debris flow descent, **o** is average precipitation intensity for the last 3 h, mm/h;

$$p_m = \begin{cases} 0 if\ (t_{10} < 0°C and t_s < 0°C) \\ \frac{2}{\pi} arctan(0.00012 q_m^{1.9}) if (t_{10} \geq 0°C or t_s \geq 0°C) \end{cases}, \quad (5)$$

where $\mathbf{p_m}$ is factor considering influence of monthly sum of precipitation on possibility of debris flow descent, $\mathbf{q_m}$ is monthly sum of precipitation, mm;

$$p_e = \begin{cases} 0 if\ (t_{10} < 0°C and t_s < 0°C) and q < 52mm \\ \frac{2}{\pi} arctan(0{,}000103 I^{5.3}) if\ (t_{10} \geq 0°C or t_s \geq 0°C) or q \geq 52mm \end{cases}, \quad (6)$$

where $\mathbf{p_e}$ is factor allowing influence of earthquake on possibility of debris flow descent, **I** is earthquake intensity on the MSK-81 scale.

$$p_{bio} = \begin{cases} \frac{2}{\pi} arctan(0.06d) if (p_q < 0.79 and p_t < 0.79 and p_m < 0.79) \\ \frac{2}{\pi} arctan(0.85d) if (p_q \geq 0.79 or p_t \geq 0.79 or p_m \geq 0.79) \end{cases}, \quad (7)$$

where $\mathbf{p_{bio}}$ is factor adverting influence of degree of expressiveness of biological precursors on possibility of debris flow descent, **d** is degree of expressiveness of biological precursors (Table 1).

Table 1. Description of degrees of expressiveness of biological precursors

Distinguishing features of abnormal behavior of animals	d
Abnormal behavior of individual specimens (no more than 5 of each species or any specimen of only 1 species).	1
Abnormal behavior of 2 or 3 species, wherein no less 6 specimens of these species demonstrate such conduct.	2
Abnormal behavior of more than 3 species, wherein no less 6 specimens of these species demonstrate such conduct.	3

If abnormal condition of people is observed, **d** is increased by 1.
After that, the following parameters are computed.

$$p_q^* = p_q^{1-0.22p_{oq}-0.07p_e-0.04p_{bio}-0.16p_t-0.07p_{t10}-0.06p_m}, \quad (8)$$

where $\mathbf{p_q^*}$ is factor adverting influence of daily precipitation amount on possibility of debris flow descent with respect to values of $\mathbf{p_{oq}}$, $\mathbf{p_e}$, $\mathbf{p_m}$, $\mathbf{p_{bio}}$, $\mathbf{p_t}$, $\mathbf{p_{t10}}$;

$$p_{oq}^* = p_{oq}^{1-0.13p_t-0.05p_e-0.06p_{bio}-0.16p_q-0.04p_{t10}-0.07p_m}, \quad (9)$$

where $\mathbf{p_{oq}}^*$ is factor allowing influence of average precipitation intensity for the last 3 h on possibility of debris flow descent with consideration to values of $\mathbf{p_q}$, $\mathbf{p_e}$, $\mathbf{p_m}$, $\mathbf{p_{bio}}$, $\mathbf{p_t}$, $\mathbf{p_{t10}}$;

$$p_e^* = p_e^{1-0.07p_t-0.01p_{oq}-0.19p_{bio}-0.02p_q-0.02p_{t10}-0.04p_m}, \tag{10}$$

where $\mathbf{p_e}^*$ – is factor allowing influence of eventual earthquake on possibility of debris flow descent with reference to values of $\mathbf{p_q}$, $\mathbf{p_i}$, $\mathbf{p_m}$, $\mathbf{p_{bio}}$, $\mathbf{p_t}$, $\mathbf{p_{t10}}$;

$$p_t^* = p_t^{1-0.12p_q-0.15p_m-0.03p_{bio}-0.09p_{oq}-0.04p_e-0.08p_{t10}}, \tag{11}$$

where $\mathbf{p_t}^*$ is factor making allowance for influence of sum of average daily temperatures after average daily temperature to exceed zero on possibility of debris flow descent with regard to values of $\mathbf{p_q}$, $\mathbf{p_e}$, $\mathbf{p_m}$, $\mathbf{p_{bio}}$, $\mathbf{p_{oq}}$, $\mathbf{p_{t10}}$;

$$p_{bio}^* = p_{bio}^{1-0.21p_e-0.07p_q-0.04p_m-0.03p_i-0.08p_t-0.02p_{t10}}, \tag{12}$$

where $\mathbf{p_{bio}}^*$ is factor making allowance for influence of biological precursors on possibility of debris flow descent in terms of values of $\mathbf{p_q}$, $\mathbf{p_e}$, $\mathbf{p_m}$, $\mathbf{p_t}$, $\mathbf{p_i}$, $\mathbf{p_{t10}}$;

$$p_m^* = p_m^{1-0.11p_q-0.1p_{oq}-0.12p_e-0.04p_{bio}-0.07p_t-0.01p_{t10}}, \tag{13}$$

where $\mathbf{p_m}^*$ – is factor considering influence of monthly sum of precipitation on possibility of debris flow descent with an eye toward values of $\mathbf{p_q}$, $\mathbf{p_e}$, $\mathbf{p_i}$, $\mathbf{p_{bio}}$, $\mathbf{p_t}$, $\mathbf{p_{t10}}$;

$$p_{t10}^* = p_m^{1-0.1p_q-0.04p_{oq}-0.02p_e-0.09p_{bio}-0.11p_t-0.03p_m}, \tag{14}$$

where $\mathbf{p_{t10}}^*$ is factor taking into account influence of average temperature for the last 10 days on possibility of debris flow descent in the face of values of $\mathbf{p_q}$, $\mathbf{p_e}$, $\mathbf{p_{oq}}$, $\mathbf{p_{bio}}$, $\mathbf{p_t}$, $\mathbf{p_m}$.

Degree of membership of conditions to debris flow descent situation is calculated as

$$p_s = \left[0.79 + 0.21\frac{2}{\pi}arctan(1.22d + 0.9q)\right] \times [1 - (1 - p_{bio}^*) \cdot (1 - p_e^*) \cdot (1 - p_{oq}^*) \cdot (1 - -p_m^*) \times\times (1 - p_q^*) \cdot (1 - p_t^*) \cdot (1 - p_{t10}^*)], \tag{15}$$

where $\mathbf{p_s}$ is degree of membership of conditions in debris flow descent situation.

Hereon, code of debris flow risk is figured out:

$$j = \begin{cases} 0\ for\ p_s < 0.17 \\ 1\ for\ 0.17 \le p_s < 0.6 \\ 2\ for\ 0.6 \le p_s < 0.9 \\ 3\ for\ 0.9 \le p_s < 0.95 \\ 4\ for\ p_s \ge 0.95 \end{cases}, \tag{16}$$

where **j** is code of debris flow risk.

If **j** > 0 and **d** = 3, **j** is increased by 1.

Debris flow forecasts depending on **j** are shown in Table 2.

This mathematical support is tested with the aid of data about debris flows happened in the past. In particular, forecast of such slope processes in Tyrnyauz area is performed using information from July 15, 2000 to July 27, 2000. Data of meteorological station of "Terskol" are applied. Seismic load is not taken into account; biological precursors were observed in Nalchik. Results of analysis are shown at Table 3. Code of debris flow risk confirmed by field observations is denoted by $\mathbf{j_r}$.

Table 2. Debris flow forecast depending on **j**

j	Forecast for the next 24 h
0	No debris flow risk.
1	Debris flows of volume up to 10^4 m^3 may occur.
2	Debris flows of volume from 10^4 to 10^5 m^3 are expected.
3	Debris flows of volume from 10^5 to 10^6 m^3 are expected.
4	Debris flows of volume from 10^6 to 10^7 m^3 are expected
5	Debris flows of volume more than 10^7 m^3 are expected

Table 3. Results of analysis of debris flow risk in July 2000.

Date	t_s, °C	t_{10}, °C	t, °C	q, mm	o, mm/h	q_m, mm	d	j	j_r
15.07	16.9	14.1	771.1	0	0	0	0	2	0
16.07	18.1	14.5	789.2	0	0	0	0	2	0
17.07	20.4	15.1	809.6	5	1	5	1	3	0
18.07	17.8	15.9	837.4	0	0	5	1	3	3
19.07	16.0	16.3	853.4	0	0	5	1	3	3
20.07	17.4	17.0	870.8	0	0	5	3	3	0
21.07	20.0	17.3	890.8	0	0	5	1	4	4
22.07	18.1	17.6	908.9	0	0	5	1	3	0
23.07	20.2	18.1	929.1	0	0	5	1	3	0
24.07	17.8	18.2	946.9	0	0	5	1	3	3
25.07	17.2	18.2	964.1	0	0	5	1	3	0
26.07	16.3	18.0	980.4	5	1	10	0	3	3
27.07	17.0	17.7	997.4	0	0	10	0	3	0

According to information of Table 3, unforetold debris flows are absent. Comparably low correctness of prognoses can be explained by requirement of high level of safety. As known [*Jonkman et al, 2003*], acceptable probability of death one person for one year is 10^{-6}.

Conclusions

Mathematical modeling of physical and mechanical processes in the debris flow source area can be useful tool to increase accuracy of debris flow forecast. Efficacy of analysis is validated experimentally. Results of the calculation can be utilized for development of prediction formulae.

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